Climate Physics Chapter 3: Radiation Balance

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Planets absorb energy as light from stars and radiate energy out in to space at a rate that increases with temperature, allowing it to keep in balance, at some equilibrium temperature, when the energy gain equals the energy loss.

Blackbody Radiation

A sufficiently hot body emits light in the form of blackbody radiation:

$$B(v,T) = \frac{2hv^3}{c^2}\frac{1}{e^{\frac{hv}{kT}} - 1} \tag{1}$$

This gives the irradiance, or the energy flux per area per frequency, of the radiation emitted from the body. In this equation, v is the frequency of the emitted radiation, h is Planck's constant, c is the speed of light, k is Boltzmann's constant, and T is the temperature of the body.
Integrating this equation over every frequency gives us the total power per unit area:

$$F = \sigma T^4 \tag{2}$$

where $\sigma \approx 5.67 * 10^{-8}$ W/m^{2}K^4 is the Stefan-Boltzmann constant.

Radiation Balance of Planets

We can calculate the temperature of a planet based on a simple accounting of the energy entering and exiting the system.
A planet receives energy from its star. This energy takes the form of black body radiation leaving the photosphere of the star. This means we can write the sun's emitted energy as $4\pi r_s^2 \sigma T_s^4$, where r_s and T_s are the sun's radius and photospheric temperature. At a distance r from the star, that radiation has spread out over a sphere of surface area $4\pi r^2$, so a planet at that distance sees a total flux of $L = \sigma T_s^4 \frac{r_s}{r}$. The planet receives this energy over its cross sectional area. If the planet's radius is a, this area is πa^2, so it receives $\pi a^2 L$ energy per unit time. However, due to the planet's reflectivity, not all of that energy will be absorbed. The proportion of sunlight reflected by the planet is the planetary albedo, denoted α. A planet with albedo of 1 would reflect all light, while a planet with $\alpha = 0$ would absorb all light. Therefore, the rate of energy absorption of a planet with albedo α and radius a is $(1 - \alpha)\pi a^2 L$.
The planet also emits energy by radiating as a black body from its entire surface. The rate of energy loss is then $4\pi a^2 \sigma T^4$, where T is the planet's surface temperature. In equilibrium, the rate of energy gain must equal the energy loss, so in equilibrium our energy balance is:

$$\sigma T^4 = \frac{1}{4}(1 - \alpha)L \tag{3}$$

or, solving for the planet's surface temperature,

$$T = \frac{1}{\sqrt{2}}(1-\alpha)^{1/4}\sqrt{\frac{r_s}{r}}T_s \tag{4}$$

The factor of $\frac{1}{4}$ comes from the ratio of the planet's cross sectional area, which receives energy, and its total surface area, which radiates energy. This is because we have averaged the energy budget over the planet's surface. For a planet that doesn't receive energy evenly over its surface, for example because it has no atmosphere to transport heat, it doesn't make sense to average over the full surface. Instead, we might want to consider a point on the surface where the sun is directly overhead. Looking at the energy per unit area for that spot, we would get an energy budget of $\sigma T^4 = (1-\alpha)L_s$, without the factor of 4.

Example Problem: Energy Balance

A spherical planet at the orbit of Mercury has a nitrogen atmosphere that has no effect on IR, but is so good at mixing heat that it keeps the planet's surface isothermal. If the surface temperature is 300K, what is the albedo?
From energy balance:

$\sigma T^4 = \frac{1}{4}L(1-\alpha) = \frac{1}{4}\sigma T_{star}^4 \frac{r_{star}^2}{r^2}(1-\alpha)$ Plugging in values: $T_{star} = 5777$ K, $r_{star} = 695700$ km, $r = 57.91e6$ km, $T = 300$ Kα

Example Problem: Energy Balance

Consider a planet covered in water ice with a uniform albedo of 0.7. The same face of the planet always faces the sun, and the atmosphere has negligible greenhouse effect. Compute the solar constant needed to begin melting ice if:
a) the entire surface of the planet has the same temperature
This means we need to average the temperature over the whole planet, giving a factor of 4 between the cross-sectional area which receives radiation and the surface area that radiates OLR:

$$L = \frac{4\pi a^2 \sigma T^4}{\pi a^2(1-\alpha)} = \frac{4 * 5.67 * 10^{-8}\ \mathrm{W/m^2K^4} * (273\ \mathrm{K})^4}{(0.3)} = 4200\ \mathrm{W/m^2}$$

b) the dayside temperature is uniform but no heat is carried to the nightside
In this case we only need to average the temperature over half the surface, so we have a factor of 2 in our energy balance equation.

$$L = \frac{2\pi a^2 \sigma T^4}{\pi a^2(1-\alpha)} = \frac{2 * 5.67 * 10^{-8}\ \mathrm{W/m^2K^4} * (273\ \mathrm{K})^4}{(0.3)} = 2100\ \mathrm{W/m^2}$$

c) there is no atmosphere, so each bit of the surface is in equilibrium with the solar radiation it absorbs
We can find the maximum temperature by looking at a patch where the sun is directly overhead, and we don't have to average the temperature over the planet at all, so the area of the patch receiving the radiation is the same patch that's emitting radiation, leaving no extra factor in our energy equation.

$$L = \frac{\sigma T^4}{(1-\alpha)} = \frac{5.67 * 10^{-8}\ \mathrm{W/m^2K^4} * (273\ \mathrm{K})^4}{(0.3)} = 1050\ \mathrm{W/m^2}$$

Greenhouse effect

We call the planet's emitted outgoing radiation the OLR, for Outgoing Longwave Radiation. It's longwave because it has a longer wavelength than the incoming solar radiation, because the planet has a lower temperature than the sun. The question we want to answer next is, how does the atmosphere affect a planet's

OLR?
The atmosphere has a temperature profile that decreases with altitude, according to the moist or dry adiabat. Say we add a gas, with uniform mass constant q, that is transparent to shortwave radiation but, provided we have enough of it, will interact strongly with longwave radiation by absorbing the radiation and then emitting energy as a black body. This is the behavior of a greenhouse gas, such as $C0_2$, H_20, and CH_4.
From the hydrostatic relation, the mass of the gas is $\frac{q\Delta p_s}{g}$. Let κ be the absorption coefficient, or the mass of the gas needed to make a column of atmosphere with a base of 1 m^2 act like a black body to certain wavelengths. If $\kappa q\Delta p/g < 1$, the air is optically thin, and will not absorb all the infrared radiation. If $\kappa q\Delta p/g >> 1$, we say the air is optically thick.
We can slice a column of air into slabs of thickness Δp_1 such that, for each slab, $\kappa q\Delta p_1/g = 1$. In this case, each slab has enough of our greenhouse gas to act as a perfect black body in the longwave. This means that only the top slab, which goes from $p = 0$ at the top to $p = p_1$ at the bottom, will radiate out to space, because any radiation from underneath it will be absorbed in the black body. So the OLR of our planet is now determined entirely by the temperature of this top slab, which we calculate by finding the temperature at pressure p_1. Since the temperature of our atmosphere decreases with altitude according to the moist or dry adiabat, this new temperature T_1 is smaller than the surface temperature, so our OLR is also smaller than it would be at the surface. In order to keep the energy budget balanced, temperature will rise, increasing OLR until equilibrium is reached at a new, warmer surface temperature.

Example Problem: Greenhouse Effect

Titan absorbs 2.94 W/m^2 of solar radiation averaged over its surface. The surface temperature is 95 K. Assume the temperature profile is given by the dry adiabat for pure N_2 with a surface pressure of 1.5 bar. What is the radiating pressure?
From the dry adiabat, we have

$$T_s = \left(\frac{p_s}{p_{rad}}\right)^{\frac{R}{c_p}} T_{rad}$$

At equilibrium, the incoming radiation must equal the OLR, so we have:

$$T_{rad} = \left(\frac{S_{in}}{\sigma}\right)^{1/4} = \left(\frac{2.94\ \mathrm{W/m^2}}{5.67\cdot 10^{-8}\ \mathrm{W/m^2K^4}}\right)^{1/4} = 84.9\ \mathrm{K}$$

Solving for p_{rad}, we are left with:

$$p_{rad} = p_s\left(\frac{T_{rad}}{T_s}\right)^{c_p/R} = 1.5\cdot 10^5\ \mathrm{Pa}\left(\frac{84.9\ \mathrm{K}}{95\ \mathrm{K}}\right)^{1037/296.9} = 1.01\cdot 10^5\ \mathrm{Pa}$$

amsmath

Ice-Albedo Feedback

Since albedo is determined by surface and atmospheric conditions, a change in the temperature of the planet can change the albedo, which in turn affects the amount of energy absorbed, which changes the equilibrium temperature. An example of this sort of climate feedback loop is Ice-Albedo feedback. Because ice is highly reflective, the amount of ice on the surface affects the planet's albedo. For simplicity, we imagine a planet covered with water, that freezes to ice if the planet gets colder. We can also use a planet with land, provided there is enough precipitation for glaciers to form on land. The planet's average surface temperature is T_s, but we assume there is some global variation so that it takes longer for ice to melt in some places (such as the poles) than in others. Above some high temperature T_w, there will be no ice anywhere on the surface and the albedo will be at its lowest value α_w. On the other end, if $T_s < T_i$, the whole surface will be covered in ice, and the albedo will reach its high value α_i. Given these constants, we can describe the albedo as a function of temperature:

$$\alpha(T) = \begin{cases} \alpha_i & T \leq T_i \\ \alpha_w + (\alpha_i - \alpha_w)\frac{(T-T_w)^2}{(T_i-T_w)^2} & T_i < T < T_w \\ \alpha_w & T \geq T_w \end{cases} \tag{5}$$

This is one example of a probable albedo function; other funtions are also possible depending on how the ice is distributed on the planet.

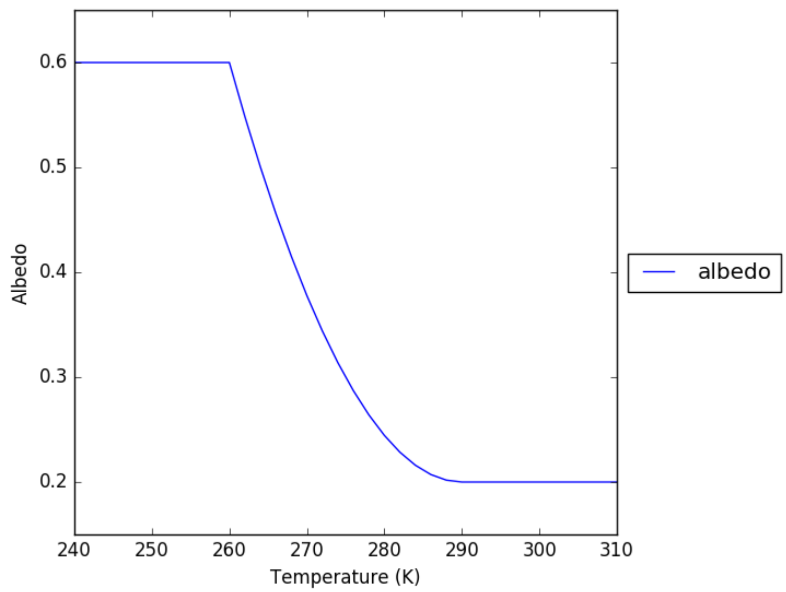

Figure 1: Graph of the albedo function over different temperatures. The cutoff temperatures are 260 and 290 K. In a climate like Earth's, albedo has a steeper slope at lower temperatures because as the temperature warms, large amounts of ice will melt quickly at the equator. As the temperature increases further, the slope decreases because small amounts of ice tend to linger at the poles.

We now investigate the affect that the albedo function has on a planet's equilibrium temperature. Ignoring for now the influence of greenhouse gases, our energy budget is:

$$(1 - \alpha(T_s))\,\frac{1}{4}L = OLR(T_s) \tag{6}$$

We plot the right hand and left hand sides of this equation on the same graph so that we can see our equilibria as intersection points.

How stable are the equilibria? We examine $E(T_s)$, the energy storage of the earth per unit area of the surface as a function of temperature. Because energy is conserved, we can say

$$\frac{dR(T_s)}{dt} = \frac{dE}{dT_s}\frac{dT_s}{dt} = G(T_s) \tag{7}$$

where $G(T_s) = \frac{1}{4}(1 - \alpha(T_s)L_s - OLR(T_s)$ is the net flux of energy entering the system. Thus:

$$\frac{dT_s}{dt} = \frac{G(T_s)}{\mu(T_s)} \tag{8}$$

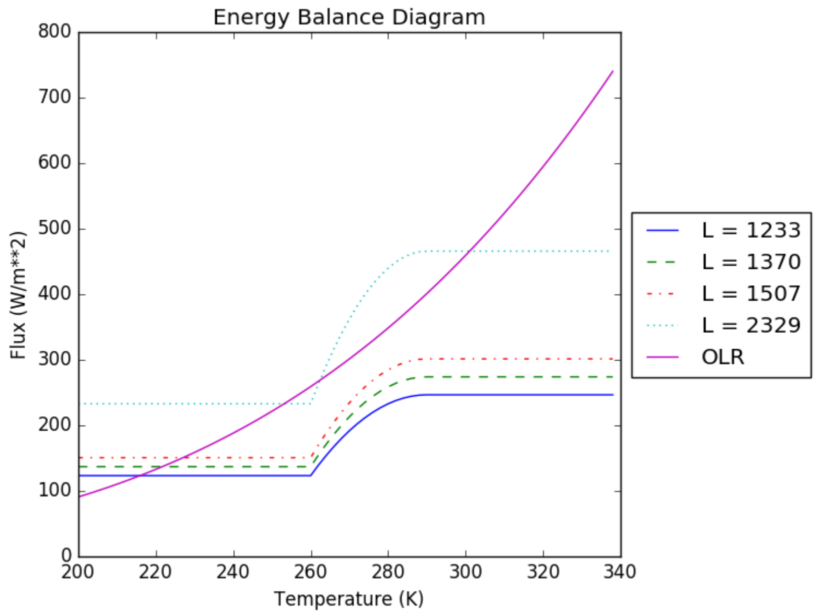

Figure 2: The calculated outgoing radiation, OLR(T), is plotted against 4 different possible values for the solar constant in the energy balance equation. Where two lines intersect, the incoming flux equals the outgoing flux and we have an equilibrium. Note that different values of L have different numbers of equilibria.

where $\mu(T_s) = \frac{dE}{dT}$ is the generalized heat capacity and is positive by assumption. Now we can look at the stability of the equilibra, which are the points where the function $G(T_s) = 0$. If the slope $\frac{dG}{dT}$ is positive at the equilibrium point, a slight increase in T causes G to increase and become positive, which means that $\frac{dT_s}{dt}$ will also be positive and T will keep increasing. In other words, since G is the net flux, if G increases this means more energy is entering the system, causing the temperature to continue increasing away from the equilibrium. Similarly, a slight decrease in T away from the equilibrium will cause T to continue decreasing, meaning that the equilibrium is not stable. On the other hand, if $\frac{dG}{dT}$ is negative at the equilibria, it will be stable, because a slight increase in T will cause $\frac{dT_s}{dt}$ to become negative and push T back to the equilibrium value, while a slight decrease in T will cause T to increase back to equilibrium. Because of the behavior and smoothness of $G(T_s)$, consecutive equilibria will alternate between being stable and unstable. A system placed between two stable equilibria will be attracted to one of the 2 and will then stay at that equilibria.

Hysteresis

We can now incorporate albedo and greenhouse gases into our temperature model.

$$(1 - \alpha(T_s))\frac{1}{4}L = \sigma T_{rad}^4 \tag{9}$$

Where T_{rad} and T_s are related by the dry adiabat: $T_s = \left(\frac{p_s}{p_{rad}}\right)^{\frac{R}{c_p}} T_{rad}$ We graph the equilibra of this equation as a function of and T_s and L, with p_{rad} held constant. One may also choose to plot T_s against p_{rad}, holding L constant.

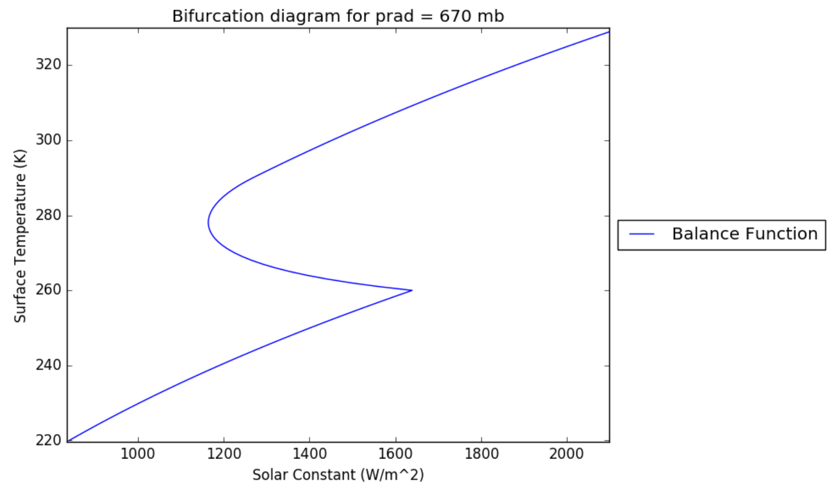

Figure 3: This is a graph of the equilibria of the radiation balance equation. The shape of this graph is because we have multiple equilibria that come in and out of existence. The segment in the middle, where the graph changes direction and has a negative slope before turning around again, represents an unstable equilibria. A system that begins on the lower, cold branch of the graph will increase along that line until the solar constant reaches around 1650 W/m^2, at which point the cold equilibrium will disappear and it will move towards the upper, hot branch of the graph. Similarly, a system that begins on the hot branch will remain there until it reaches 1150 W/m^2, at which point it will transition to the cold branch. The temperature the system will have at a solar constant between 1150 and 1650 W/m^2 depends on where it was before. This is an example of hysteresis, which is when the state of a system depends on some parameter of the system as well as on the history of variation of that parameter. Looking at this behavior, one can see how a moderate equilibrium temperature of, say, 290 K, can be quite fragile.

Climate Sensitivity, Radiative Forcing, and Feedback

Suppose the temperature of our system depends on some parameter Λ. The sensitivity of T to Λ is then $\frac{d\Lambda}{dT}$. The net flux, G, may also depend on $\Lambda : G = G(T, \Lambda)$. If we take the derivative of the equilibrium $G = 0$, we find:

$$\frac{dT}{d\Lambda} = -\frac{\frac{\partial G}{\partial \Lambda}}{\frac{\partial G}{\partial T}} \tag{10}$$

The numerator, $\frac{\partial G}{\partial \Lambda}$, is a measure of the radiative forcing associated with a change in Λ. If we change Λ by $\delta\Lambda$, G changes by $\frac{\partial G}{\partial \Lambda}\delta\Lambda$, and T must change to bring the system back into balance. The denominator, $\frac{\partial G}{\partial T}$, can be called the climate sensitivity factor, as it determines how much T_{eq} will change given some radiative forcing. For example, if we want to look at ice-albedo feedback, we can plug in our equation for G into equation 10, yielding:

$$\frac{dT}{d\Lambda} = \frac{1}{1+\Phi}\left[\frac{\frac{\partial G}{\partial \Lambda}}{\frac{\partial OLR}{\partial T}}\right] \tag{11}$$

Where

$$\Phi = \frac{1}{4}L\frac{\frac{\partial \alpha}{\partial T}}{\frac{\partial OLR}{\partial T}} \tag{12}$$

. The factor in square brackets in equation 11 is the sensitivity the system would have to Λ if unmodified by the change of albedo with temperature. The first factor determines how much the sensitivity changes due to the influence of T on albedo.

If $-1 < \Phi < 0$, the forcing produces a larger change than we would see otherwise.
If $-2 < \Phi < -1$, the feedback is so large that it reverses the sign of the response.
If $\Phi > 0$, the feedback reduces the sensitivity, producing a smaller change than we would see otherwise.
We can also generalize the above example: Say G depends on the control parameter Λ and some other parameter $\mathcal{R}$ which varies with temperature. In the example, $\mathcal{R}$ was the temperature-dependent albedo. Then, following the same line of reasoning as in the albedo example, we can write:

$$\Phi = \frac{1}{4} L \frac{\frac{\partial G}{\partial \mathcal{R}} \frac{\partial \mathcal{R}}{\partial T}}{\frac{\partial G}{\partial T}} \tag{13}$$

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Partially Absorbing Atmospheres

Usually, we can assume that blackbody radiation is in equilibrium with the surrounding matter because radiation interacts so strongly with matter. However, if some matter interacts only weakly with radiation, its absorption and emission will not look like that of an ideal black body. For the emission spectrum I, we get:

$$I(v, \hat{n}) = e(v, \hat{n}) B(v, T), \tag{14}$$

where v is the frequency of radiation, $\hat{n}$ is the direction of radiation, and T is the temperature of the matter. Here, $e(v, \hat{n})$ is the emissivity function of the matter. The net flux from a patch of the surface is then:

$$F = \bar{e} \sigma T^4 \tag{15}$$

We also define analogously the absorptivity $a(v, \hat{n})$, where v and $\hat{n}$ are the frequency and direction of the incoming radiation. Since the matter does not act like a black body, it may transmit or reflect energy, instead of absorbing it. So we say a is the ratio of the net flux absorbed by the object to the total incident flux:

$$a(v, \hat{n}) = \frac{F_{inc} - (T + R)}{F_{inc}} \tag{16}$$

where T is the transmitted flux and R is the reflected flux.
Kirchoff's law of radiation relates a and e. It states:

$$\frac{e}{a} = 1 \tag{17}$$

at each frequency for almost all materials. This does not hold for materials that absorb, store, and reemit energy, such as phosphorescent or fluorescent materials. However, for planetary materials on a large scale, we can treat the law as true.

Optically Thin Atmospheres: The Skin Temperature

Since atmosphere density approaches zero with height, we can define an outer layer of atmosphere with low enough density that it has a very low infrared emissivity. We call this the skin layer. We can suppose that it is transparent to solar radiation, and that atmospheric circulation transports very little heat to the skin, so it is heated only by IR upwelling from below. Since it absorbs very little of this radiation, the upwelling radiation is very similar to the OLR. We can then write the energy balance as just from infrared radiation:

$$2 e_{IR} \sigma T_{skin}^4 = e_{IR} OLR \tag{18}$$

The factor of 2 on the left is because the skin radiates from the top and from the bottom. Since $\frac{e}{a} = 1$, we have replaced a with e on the right hand side. Solving for T_{skin}:

$$T_{skin} = \frac{1}{2^{1/4}} \left(\frac{OLR}{\sigma} \right)^{1/4} = \frac{1}{2^{1/4}} T_{rad} \tag{19}$$

So T_{skin} is colder than the natural radiating temperature T_{rad} by a factor of $\frac{1}{2^{1/4}}$. A layer that has low emissivity and absorptivity in some wavelength band is said to be optically thin in that band. What about the case where the whole atmosphere is optically thin? We can also assume that the atmosphere doesn't block incoming solar light. Let the incident solar flux per unit area be denoted S. Since the atmosphere is so thin, we can say $p_{rad} = p_s$, that is, the atmosphere does not affect the temperature of the surface. The skin layer in this case extends all the way to the ground. If we assume no heat convection, whole atmosphere is then isothermal, so:

$$T_{atmosphere} = T_{skin} = 2^{-1/4} T_s \tag{20}$$

This tells us that the surface temperature is warmer than the air, which means heat will begin to non-radiatively conduct into the air, causing convection in the atmosphere. As a result, the atmosphere's temperature profile will follow the adiabat from the surface upward until it reaches T_{skin}, at which point it will stabilize, and the temperature will remain at T_{skin} as we keep moving up. We use this boundary to define the tropopause. Below it, where temperature decreases with altitude, we have the troposphere, and the isothermal region above it is the stratosphere. We can find the pressure of the tropopause by requiring that the isosphere temperature T_{skin} from equation 20 equals the pressure given by the adiabat starting at the surface. Using the dry adiabat, this yields:

$$\frac{p_{trop}}{p_s} = 2^{\left(-\frac{c_p}{4R}\right)} \tag{21}$$

where p_{trop} is the tropopause pressure. Usually, however, stratospheres aren't isothermal because they also absorb some solar radiation, which we have not accounted for, causing their temperature to increase with altitude. To account for this, let e_{IR} be the skin's infrared emissivity, and a_{sw} be the skin's shortwave absorptivity, so that the sunlight absorbed is Sa_{sw}. e and a do not have to be equal because they are at different wavelengths. If the portion of sunlight being absorbed is absorbed fully, which is typical. we don't have to account for light reflecting off the surface. The energy balance for the skin layer is then:

$$2e_{IR}\sigma T_{skin}^4 = e_{IR} OLR + a_{sw} S \tag{22}$$

. Solving for skin temperature we have,

$$T = T_{sk} \left(1 + \frac{a_{sw}}{e_{IR}} \frac{S}{OLR} \right)^{1/4} \tag{23}$$

where T_{sk} is the temperature the skin would have without solar radiation, as in equation 19. This shows that solar absorption increases the temperature of the skin layer, and that the temperature increases as the ratio $\frac{S}{OLR}$ increases.

If a_{sw} is small, we can divide the skin into a number of sublayers, and use the same techniques to determine the temperature of each sublayer. This tells us that the temperature of the stratosphere will increase with height if absorption increases with height. On Earth, temperature in the stratosphere increases sharply with height. The solar absorption that causes this is due to the ozone, which strongly absorbs ultraviolet light, shielding us from dangerous UV rays.