Solitons and Breather type solutions of some nonlinear equations by the Sine-Cosine method.

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Abstract: In our recent work, we study a few nonlinear time evolution equations by the sine-cosine method and obtained a variety of generalized solitary and periodic solutions with distinct physical structures. The solutions include periodic solutions, soliton solutions, symmetric periodic soliton solutions, double periodic solutions, multiple soliton solutions, breather solutions, and kink type solutions.
KEY WORDS: The Sine-Cosine method; The Zoomeran equation; The Hirota-Ramani equation;The Zarkhov-Kuznetsov-benjamin-Bona-Mohanty equation; The Konno-Oono equation.
[1]

1 Introduction

In the past decade, with the development of new mathematical formulations and approximations, researchers are paying much interest in the solution of nonlinear systems by the implementation of various tools and techniques. The approach of solving these systems is mainly governed by Nonlinear Evolution Equations (NLEEs) along with others. However, these equations are being implemented directly or indirectly in Applied Physics and Mathematics, Population Growth Dynamic models, Nonlinear Optics, and many more. Hence the solution to NLEEs is drawing a great deal of attention in the research society in these recent years. The analytical solution of NLEEs is of paramount importance in solving mathematical and physical models. The process flow of finding an exact solution to NLEEs involves many steps. First of all, a test function has to be constructed. Using suitable mathematical conditions and approximations, the analytical solution of the function can be developed further which finds its real-time application in different models. As a result of an extensive literature survey, it has been observed that the solution to NLEEs can be performed by various methods. Some important methods include Variational Iteration method [1, 2, 3], The $(\frac{G'}{G}, \frac{1}{G})$-method [4, 5], Truncated Painlevae expansion method [6], Adomionos Decomposition method [7, 8], The $(\frac{G'}{G})$ method [9, 10, 11], The Jacobi Elliptic Functions method [12], The Sine-cosine method [13, 14] and many more. Among these methods, We are motivated to implement the sine-cosine method which is based on solving a non-linear partial differential equation with less computational work, which provides nearly exact solution to NLEEs. The following models are taken to investigate with their graphical analysis, they are listed below.

1. The Zoomeran equation;

[1] Asterisk/'*' stands for corresponding author

2. The Hirota-Ramani equation;

3. The Zarkhov-Kuznetsov-benjamin-Bona-Mohanty equation;

4. The Konno-Oono equation.

2 Analysis of the Sine-Cosine method

We start our discussion with NLEE, which can be specified as a combination of different order dependent and independent terms and their partial derivative.

$$S(f, f_t, f_x, f_{tt}, f_{xt}, f_{xx}, ...) = 0, \tag{1}$$

Where $f = f(x, t)$ being an trial function.
The major footsteps of the Sine-Cosine method are illustrates as below:
Step 1:To generalize the exact traveling wave solution of eq.(1), we are taking the wave variable as below;

$$(\phi) = (x - ct), \tag{2}$$

therefore

$$f(x, t) = f(\phi), \tag{3}$$

The resulting following changes can be noted:

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \phi}, \quad \frac{\partial^2}{\partial t^2} = c^2\frac{\partial^2}{\partial \phi^2}, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial \phi}, \quad \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \phi^2}, \tag{4}$$

and can be extended to higher orders.Now eq.(1) we can be written as,

$$s(f, f_\phi, f_{\phi\phi}, f_{\phi\phi\phi}, ...) = 0, \tag{5}$$

Here f_ϕ denotes $\frac{df}{d\phi}$.
We can integrate the obtained ODE (5) as many times to get a comparatively simpler equation, for simplicity equate the constants of integration to zero.

Step 2: The sine-cosine method uses the following trigonometric trial solutions of many nonlinear equations,the (cos) form can be expressed as:

$$f(x, t) = \lambda \cos^\beta(\mu\phi), [\phi]<\frac{\pi}{2\mu}, \tag{6}$$

and the (sin) form expressed as:

$$f(x, t) = \lambda \sin^\beta(\mu\phi), [\phi]<\frac{\pi}{2\mu}, \tag{7}$$

where λ,μ, and β are integers which will be determined later, μ and c are the wave number and the wave speed, respectively. Eq.(6) can be generalized

$$f(\phi) = \lambda \cos^\beta(\mu\phi), \tag{8}$$

$$(f^n)_{\phi\phi} = -n^2\mu^2\beta^2\lambda^n \cos^n \beta(\mu\phi) + n\mu^2\lambda^n\beta(n\beta 1)\cos^{n\beta-2}(\mu\phi), \tag{9}$$

and for eq.(7) we use

$$f(\phi) = \lambda \sin^\beta(\mu\phi), \tag{10}$$

$$(f^n)_{\phi\phi} = -n^2\mu^2\beta^2\lambda^n \sin^n \beta(\mu\phi) + n\mu^2\lambda^n\beta(n\beta 1)\sin^n \beta 2(\mu\phi), \tag{11}$$

Step 3:After substituting eq.(6) or eq.(7) in eq.(5), we will get an equation of $\cos^\beta(\mu\phi)$ or $\sin^\beta(\mu\phi)$ forms. Rationalizing the resulted equation for λ,β, and μ as follows:

1. At first balance the exponents by homogeneous balance to determine β;
2. Secondly collect the coefficients in terms same power of $\cos^\beta(\mu\phi)$ or $\sin^\beta(\mu\phi)$, and equate them zero separately.
3. Finally λ, μ, and β can be calculated from the algebraic systems.

3 Application of Sine-Cosine Method.

The Sine-cosine method can be implemented to disclose more traveling wave solutions of a class of NLEEs.

3.1 The Zoomeran Equation

The Zoomeran equation can be written as

$$\left(\frac{f_{xy}}{f}\right)_{tt} - \left(\frac{f_{xy}}{f}\right)_{xx} + 2\left(f^2\right)_{xt} = 0 \tag{12}$$

In order to solve eq.(12) by the sine-cosine method, we use the wave transformation $f(x,t) = f(\phi)$ with wave variable $\phi = (x - cy - wt)$ eq.(12) takes the form of an ODE.

$$c(1-w^2)f'' + 2wf^3 - Rf = 0 \tag{13}$$

Here R stands for integration constant,by considering the (cos) term solution,

$$f(\phi) = \lambda \cos^\beta(\mu\phi), \tag{14}$$

$$(f)_\phi = -\mu\beta\lambda \sin^\beta(\mu\phi)\cos^{\beta-1}(\mu\phi), \tag{15}$$

$$(f)_{\phi\phi} = -\mu^2\beta^2\lambda\cos^\beta(\mu\phi) + \mu^2\lambda\beta(\beta-1)\cos^{\beta-2}(\mu\phi), \tag{16}$$

substituting these values in eq. (12), we will get

$$R\lambda\cos^\beta \mu\xi + 2w\lambda^{3\beta}\mu\xi + c(1-w^2)(\lambda\mu^2\beta^2\cos^\beta(\mu\xi) - \lambda\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\xi)) = 0. \tag{17}$$

Collecting the coefficients in terms same power of $\cos^\beta(\mu\phi)$, and equating them zero separately. The following set of algebraic systems can be realized:

$$(\beta - 1) \neq 0, \tag{18}$$

$$(\beta - 2) = 3\beta, \tag{19}$$

$$R = -c(1 - w^2)\mu^2\beta^2, \tag{20}$$

$$2w\lambda^2 = c(1 - w^2)\mu^2\beta(\beta - 1), \tag{21}$$

after solving these we will get,

$$\beta = -1, \mu = \sqrt{\frac{-R}{c(1 - w^2)}}, \lambda = \sqrt{-2R}, \tag{22}$$

the traveling wave solutions will be, for $w < 0$

$$f_1(x, t) = \sqrt{-2R}\sec\left[\sqrt{\frac{-R}{c(1 - w^2)}}(x - ct)\right], w < 0, \tag{23}$$

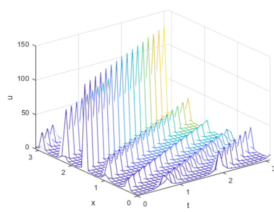

Figure 1: (
Multiple periodic soliton solution corresponding to f_1,when $w > 0$ with $c = 1, w = 2$)

$$f_2(x, t) = \sqrt{-2R}\csc\left[\sqrt{\frac{-R}{c(1 - w^2)}}(x - ct)\right], w < 0, \tag{24}$$

Similarly for $w > 0$

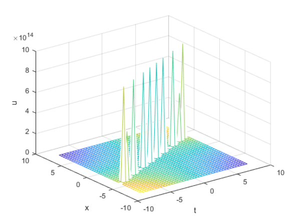

Figure 2: (
Multiple soliton solution corresponding to f_2,when $w > 0$ with $c = 1, w = 2$)

$$f_3(x, t) = \sqrt{-2R}\,\mathrm{sech}\left[\sqrt{\frac{R}{c(w^2 - 1)}}(x - ct)\right], w > 0, \tag{25}$$

$$f_4(x, t) = \sqrt{-2R}\,\mathrm{csch}\left[\sqrt{\frac{R}{c(w^2 - 1)}}(x - ct)\right], w > 0, \tag{26}$$

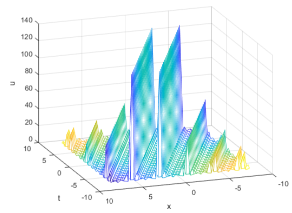

Figure 3: (
Symmetric periodic multiple soliton solution corresponding to f_3,when $w < 0$ with $c = 1, w = -2$)

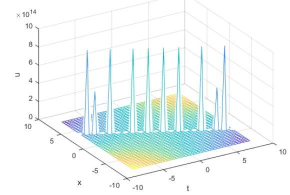

Figure 4: (
Multiple soliton solution corresponding to f_4,when $w < 0$ with $c = 1, w = -2$)

3.2 Hirota-Ramani Equation

Now we consider Hirota-ramani equation,

$$f_t - f_{xxt} + af_x(1 - f_t) = 0, \tag{27}$$

Here $a \neq 0$ and 'a' is an integer, In order to solve eq.(27) by the sine-cosine method, we use the wave transformation $f(x,t) = f(\phi)$ with wave variable $\phi = (x - ct)$ eq.(27) takes the form of an ordinary differential equation.

$$(a - w)f' + wf''' + wa(f')^2 = 0, \tag{28}$$

For simplicity let $f' = g$

$$(a - w)g + wg'' + wa(g)^2 = 0, \tag{29}$$

and by taking (cos) term trial solution

$$f(\phi) = \lambda \cos^{\beta}(\mu\phi), \tag{30}$$

$$(g)_{\phi} = -\mu\beta\lambda \sin^{\beta}(\mu\phi) \cos^{\beta-1}(\mu\phi), \tag{31}$$

$$(g)_{\phi\phi} = -\mu^2\beta^2\lambda \cos^{\beta}(\mu\phi) + \mu^2\lambda\beta(\beta - 1) \cos^{\beta-2}(\mu\phi), \tag{32}$$

substituting these values in eq.(27), we will get

$$(a - w)\lambda \cos^{\beta} \mu\phi + wa\lambda^2 \cos^{2\beta} \mu\phi + w\lambda\mu^2\beta(\beta - 1) \cos^{\beta-2}(\mu\phi) - \lambda\mu^2\beta^2 \cos^{\beta}(\mu\phi) = 0, \tag{33}$$

Collecting the coefficients in terms same power of $\cos^{\beta}(\mu\phi)$, and equating them zero separately. The following set of algebraic systems can be realized:

$$(\beta - 1) \neq 0, \tag{34}$$

$$(\beta - 2) = 2\beta, \tag{35}$$

$$(a - c) = \mu^2 \beta^2, \tag{36}$$

$$a\lambda = \mu^2 \beta(\beta - 1), \tag{37}$$

after solving these we will get,

$$\beta = -2, \mu = \pm\frac{1}{2}\sqrt{a-c}, \lambda = \frac{3(a-c)}{2a}, \tag{38}$$

the travelling wave solutions will be, for $a > c$

$$f_5(x,t) = \int g_5(\phi)d\phi = \int \frac{3(a-c)}{2a} \sec^2\left[\pm\frac{1}{2}\sqrt{a-c}(x-ct)\right] d\phi, a > c, \tag{39}$$

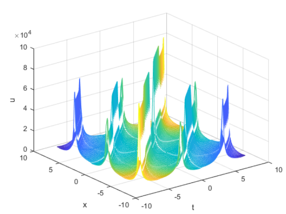

Figure 5: (
Breather solution corresponding to f_5, when $a > c$ with $c = 1, a = 2$)

$$f_6(x,t) = \int g_6(\phi)d\phi = \int \frac{3(a-c)}{2a} \csc^2\left[\pm\frac{1}{2}\sqrt{a-c}(x-ct)\right] d\phi, a > c, \tag{40}$$

Similarly for $a < c$

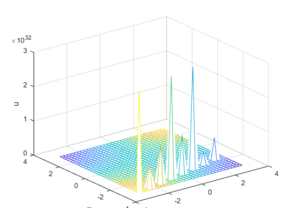

Figure 6: (
Multiple soliton solution corresponding to f_6, when $a > c$ with $c = 2, a = 3$)

$$f_7(x,t) = \int g_7(\phi)d\phi = \int \frac{3(c-a)}{2a} \operatorname{sech}^2\left[\pm\frac{1}{2}\sqrt{c-a}(x-ct)\right] d\phi, a < c, \tag{41}$$

$$f_8(x,t) = \int g_8(\phi)d\phi = \int i\frac{3(c-a)}{2a} \operatorname{csch}^2\left[\pm\frac{1}{2}\sqrt{c-a}(x-ct)\right] d\phi, a < c, i = \sqrt{-1}, \tag{42}$$

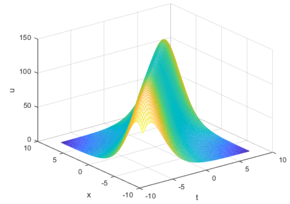

Figure 7: (
Breather solution corresponding to f_7,when $a < c$ with $c = 1, a = -1$)

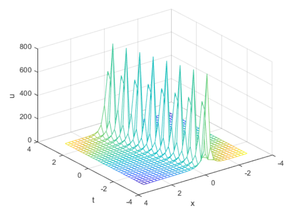

Figure 8: (
Bidirectional soliton solution corresponding to f_8,when $a < c$ with $c = 2, a = -2$)

3.3 The Zarkhov-Kuznetsov-benjamin-Bona-Mohanty Equation

Now we will consider the Zarkhov-kuznetsov-benjamin-bona-mohanty equation,

$$f_t + f_x - 2aff_x - bf_{xxt} = 0, \tag{43}$$

In order to solve eq.(43) by the sine cosine method, we use the wave transformation $f(x,t) = f(\phi)$ with wave variable $\phi = (x - ct)$ eq.(43) takes the form of an ordinary differential equation.

$$(1-c)f - af^2 + bcf'' = 0 \tag{44}$$

now consider the (cos) term trial solution as previously

$$f(\phi) = \lambda \cos^{\beta}(\mu\phi), \tag{45}$$

$$(f)_{\phi} = -\mu\beta\lambda \sin^{\beta}(\mu\phi)\cos^{\beta-1}(\mu\phi), \tag{46}$$

$$(f)_{\phi\phi} = -\mu^2\beta^2\lambda\cos^{\beta}(\mu\phi) + \mu^2\lambda\beta(\beta-1)\cos^{\beta-2}(\mu\phi), \tag{47}$$

substituting these values in eq.(43), we will get

$$(1-c)\lambda\cos^{\beta}\mu\phi - a\lambda^2\cos^{2\beta}\mu\phi + bc\lambda\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\phi) - bc\lambda\mu^2\beta^2\cos^{\beta}(\mu\phi) = 0, \tag{48}$$

Collecting the coefficients in terms same power of $\cos^{\beta}(\mu\phi)$, and equating them zero separately. The following set of algebraic systems can be realized:

$$(\beta - 1) \neq 0, \tag{49}$$

$$(\beta - 2) = 2\beta, \tag{50}$$

$$(1-c) = bc\mu^2\beta^2, \tag{51}$$

$$a\lambda = bc\mu^2\beta(\beta-1), \tag{52}$$

after solving these we will get,

$$\beta = -2, \mu = \pm\frac{1}{2}\sqrt{\frac{1-c}{bc}}, \lambda = \frac{1.5(1-c)}{a}, \tag{53}$$

the traveling wave solutions will be, for $a, b, c > 0$

$$f_9(x,t) = \frac{1.5(1-c)}{a}\sec^2\left[\pm\frac{1}{2}\sqrt{\frac{1-c}{bc}}(x-ct)\right], a, b, c > 0, \tag{54}$$

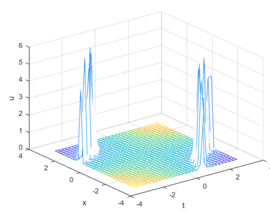

Figure 9: (
Multiple soliton solution corresponding to f_9,when $b, c > a$ with $c = 2, a = -1, b = 2$)

$$f_{10}(x,t) = \frac{1.5(1-c)}{a}\csc^2\left[\pm\frac{1}{2}\sqrt{\frac{1-c}{bc}}(x-ct)\right], a, b, c > 0, \tag{55}$$

Similarly for $a, b, c > 0$

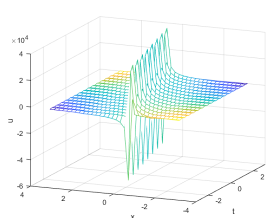

Figure 10: (
Kink and soliton solution corresponding to f_{10},when $b, c > a$ with $c = 2, a = -1, b = 2$)

$$f_{11}(x,t) = \frac{1.5(1-c)}{a}\operatorname{sech}^2\left[\pm\frac{1}{2}\sqrt{\frac{1-c}{bc}}(x-ct)\right], a, b, c > 0, \tag{56}$$

$$f_{12}(x,t) = \frac{1.5(1-c)}{a}\operatorname{csch}^2\left[\pm\frac{1}{2}\sqrt{\frac{1-c}{bc}}(x-ct)\right], a, b, c > 0, \tag{57}$$

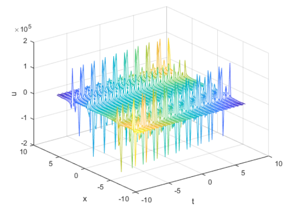

Figure 11: (
Breather type soliton solution corresponding to f_{11}, when $a, b, c > 0$ with $c = 3, a = 1, b = 2$)

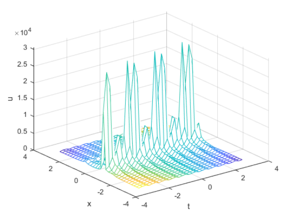

Figure 12: (
Bidirectional soliton solution corresponding to f_{12}, when $a, b, c > 0$ with $c = 3, a = 1, b = 2$)

3.4 The Konno-Oono equation

Now we consider The Konno-oono equation.

$$f_{xt} - 2fg = 0, g_t + 2gg_x = 0, \tag{58}$$

In order to solve eq.(58) by the sine cosine method, we use the wave transformation $f(x,t) = f(\phi), g(x,t) = g(\phi)$ with wave variable $\phi = (x - ct)$ eq.(58) takes the form of an ODE.

$$-cf'' - 2fg = 0, \tag{59}$$

$$-cf' + 2ff' = 0, \tag{60}$$

After integrating equation eq.(60) with respect to (ϕ) we obtain

$$g = \frac{1}{c}(f^2 + d) \tag{61}$$

Where d is an integral constant.Substituting eq.(61) into the eq.(59), we obtain

$$c^2 f'' + fd + 2f^3 = 0, \tag{62}$$

Similarly considering the (cos) term trial solution

$$f(\phi) = \lambda \cos^{\beta}(\mu\phi), \tag{63}$$

$$(f)_{\phi} = -\mu\beta\lambda \sin^{\beta}(\mu\phi) \cos^{\beta-1}(\mu\phi), \tag{64}$$

$$(f)_{\phi\phi} = -\mu^2\beta^2\lambda \cos^{\beta}(\mu\phi) + \mu^2\lambda\beta(\beta - 1) \cos^{\beta-2}(\mu\phi), \tag{65}$$

substituting these values, we will get

$$2d\lambda\cos^{\beta}\mu\phi + 2\lambda^3\cos^{3\beta}\mu\phi + c^2\lambda\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\phi) - c^2\lambda\mu^2\beta^2\cos^{\beta}(\mu\phi) = 0, \quad (66)$$

Collecting the coefficients in terms same power of $\cos^{\beta}(\mu\phi)$, and equating them zero separately. The following set of algebraic systems can be realized:

$$(\beta - 1) \neq 0, \quad (67)$$

$$(\beta - 2) = 3\beta, \quad (68)$$

$$2d = -c^2\mu^2\beta^2, \quad (69)$$

$$2\lambda^2 = c^2\mu^2\beta(\beta-1), \quad (70)$$

after solving these we will get,

$$\beta = -1, \mu = \sqrt{\frac{-2d}{c^2}}, \lambda = \mp 2\sqrt{d}, \quad (71)$$

the travelling wave solutions will be, for $d > 0$

$$f_{13}(x,t) = 2\sqrt{d}\sec\left[\sqrt{\frac{-2d}{c^2}}(x-ct)\right], d > 0, \quad (72)$$

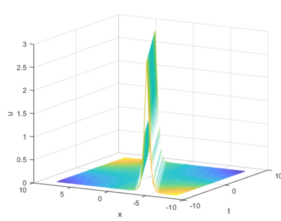

Figure 13: (
Singular kink soliton solution corresponding to f_{13}, when$d > 0$ with $c = 1, d = 2$)

$$f_{14}(x,t) = 2\sqrt{d}\csc\left[\sqrt{\frac{-2d}{c^2}}(x-ct)\right], d > 0, \quad (73)$$

Similarly for $d < 0$

$$f_{15}(x,t) = 2\sqrt{d}\,\mathrm{sech}\left[\sqrt{\frac{-2d}{c^2}}(x-ct)\right], d < 0, \quad (74)$$

$$f_{16}(x,t) = 2\sqrt{d}\,\mathrm{csch}\left[\sqrt{\frac{-2d}{c^2}}(x-ct)\right], d < 0, \quad (75)$$

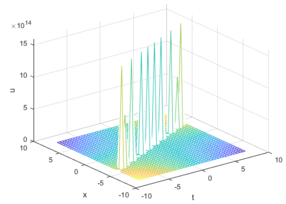

Figure 14: (
Multiple soliton solution corresponding to f_{14}, when$d > 0$ with $c = 1, d = 2$)

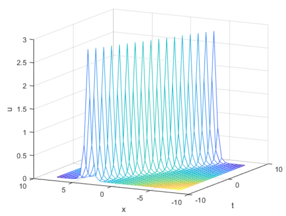

Figure 15: (
Multiple soliton solution corresponding to f_{15}, when $d < 0$ with $c = 1, d = -2$)

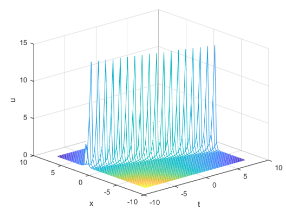

Figure 16: (
Multiple soliton solution corresponding to f_{16}, when $d < 0$ with $c = 1, d = -2$)

4 Conclusion

1. The sine-cosine method has been presented in detail and applied to generate multiple exact traveling wave solutions involving some free parameters.
2. It is fascinating to notice that the general traveling wave solutions give a different type of soliton solutions like periodic soliton solution, symmetric periodic soliton solutions, breather type solutions, singular kink solutions and periodic solutions under some special conditions.
3. The plots are very clear to understand the nature of the solutions.
4. Through there is a class of different methods available to handle NLEEs, our method gives better solutions with less computational work.
5. The advantage of this method is more effective, reliable, compact, concise.
6. The authors encouraging the research community it will be a good option to find traveling wave solution of any new NLEEs.
7. Further any modification of the method can give more solutions, It can be taken as a challenge.

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