Supporting Information for Rapid ductile strain localization due to thermal runaway"

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Introduction

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Text S1: Derivation of nondimensionalized energy conservation equation

Here, we present the detailed derivation of the nondimensionalized energy equation (eq. (28)), which reads in dimensional form as follows:

$$\rho\, C_p \frac{dT}{dt} = \lambda \frac{\partial^2 T}{\partial x^2} + \tau \dot{\varepsilon}_{\mathrm{vi}}. \tag{S1}$$

To derive the nondimensional form of equation (S1), we have to define the characteristic scales for time t_c, temperature T_c, stress τ_c and length l_c. This allows us to express the dimensional values of these quantities as

$$T = T_\mathrm{c}\, T', x = l_\mathrm{c}\, x', t = t_\mathrm{c}\, t', \tau = \tau_\mathrm{c}\, \tau'. \tag{S2}$$

Using these expressions, we can then rewrite eq. (S1) to yield

$$\rho\, C_p \frac{T_\mathrm{c}}{t_\mathrm{c}} \frac{\partial T'}{\partial t'} = \lambda \frac{T_\mathrm{c}}{l_\mathrm{c}^2} \frac{\partial^2 T'}{\partial x'^2} + \tau_\mathrm{c} \tau'\, \dot{\varepsilon}_{\mathrm{vi}}, \tag{S3}$$

which can be rearranged to

$$\frac{\partial T'}{\partial t'} = t_\mathrm{c} \frac{\kappa}{l_\mathrm{c}^2} \frac{\partial^2 T'}{\partial x'^2} + \frac{t_\mathrm{c}}{T_\mathrm{c}} \frac{1}{\rho\, C_p} \tau_\mathrm{c} \tau'\, \dot{\varepsilon}_{\mathrm{vi}}. \tag{S4}$$

Based on the observations in section 3.1, we make the following assumptions to define the characteristic scales for time, temperature, stress and length:

(i) The occurrence of thermal runaway is governed by the conditions when stress relaxation starts (black crosses in Figure 2).

(ii) Stress relaxation starts after the transition from elastic loading to dislocation creep or after the transition from low-temperature to dislocation creep.

(iii) There is no significant amount of temperature change during elastic loading.

(iv) Temperature change before stress relaxation is homogeneous in the model domain.

We therefore define the characteristic stress as

$$\tau_{\mathrm{c}} = \min\left(\tau_{\mathrm{dis},0}, \sigma_{\mathrm{b}}\right), \tag{S5}$$

where σ_{b} is the back stress of LTP which describes the transition from LTP to dislocation creep. $\tau_{\mathrm{dis},0}$ is the steady state stress in dislocation creep at the initial temperature which is defined as

$$\tau_{\mathrm{dis},0} = \left(\frac{\dot{\varepsilon}_{\mathrm{an}}}{\omega_0\, A_{\mathrm{dis}}} e^{\frac{Q_{\mathrm{dis}}}{T_0}}\right)^{\frac{1}{n}}. \tag{S6}$$

$\dot{\varepsilon}_{\mathrm{an}}$ is the steady state viscous strain rate in the center of the anomaly. To determine this value, we consider deformation to be fully accommodated by dislocation creep:

$$\dot{\varepsilon}_{\mathrm{bg}}\, L = \int_{-L/2}^{L/2} \dot{\varepsilon}_{\mathrm{dis}}\, dx, \tag{S7}$$

which extends to

$$\dot{\varepsilon}_{\mathrm{bg}}\, L = \int_{-L/2}^{L/2} \omega(x) A_{\mathrm{dis}}\, e^{-\frac{Q_{\mathrm{dis}}}{T}}\, \tau^n\, dx. \tag{S8}$$

Based on assumption (iv), this integral can be expressed as

$$\dot{\varepsilon}_{\mathrm{bg}}\, L = A_{\mathrm{dis}}\, e^{-\frac{Q_{\mathrm{dis}}}{T}}\, \tau^n \int_{-L/2}^{L/2} \omega(x)\, dx. \tag{S9}$$

$\omega(x)$ is given by equation (1) and computing the integral yields

$$\int_{-L/2}^{L/2} \omega(x)\, dx = \int_{-L/2}^{L/2} 1\, dx + \int_{-L/2}^{L/2} (\omega_0 - 1)\, e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2} dx \tag{S10}$$

$$= x\Big|_{-L/2}^{L/2} + \sigma \frac{\sqrt{2\pi}}{2} (\omega_0 - 1) \operatorname{erf}\left(\frac{x}{\sqrt{2}\sigma}\right)\Big|_{-L/2}^{L/2} \tag{S11}$$

$$= L + \sigma\sqrt{2\pi}\, (\omega_0 - 1) \operatorname{erf}\left(\frac{L}{\sigma\sqrt{8}}\right). \tag{S12}$$

Substituting equation (S12) into equation (S9) and dividing by L yields

$$\dot{\varepsilon}_{bg} = A_{\text{dis}}\, e^{-\frac{Q_{\text{dis}}}{T}} \tau^n \left[1 + \frac{\sigma}{L} \sqrt{2\pi}\; (\omega_0 - 1) \operatorname{erf}\left(\frac{L}{\sigma\sqrt{8}}\right)\right]. \tag{S13}$$

Here, we define:

$$\dot{\varepsilon}_{\text{an}} = \omega_0\, A_{\text{dis}}\, e^{-\frac{Q_{\text{dis}}}{T}}\, \tau^n. \tag{S14}$$

Substituting equation (S14) into equation (S13) yields

$$\dot{\varepsilon}_{bg} = \dot{\varepsilon}_{\text{an}}\, f_{\text{an}}, \tag{S15}$$

where f_{an} is a factor describing shape, size and strength of the anomaly:

$$f_{an} = \frac{1}{\omega_0} + \frac{\sigma}{L}\sqrt{2\pi}\, \frac{\omega_0 - 1}{\omega_0}\; \operatorname{erf}\left(\frac{L}{\sigma\sqrt{8}}\right). \tag{S16}$$

Equations (S14) and (S15) are only applicable when dislocation creep is the dominant deformation mechanism. The steady state stress in dislocation creep given in equation (S6) can hence be written as

$$\tau_{\text{dis},0} = \left(\frac{\dot{\varepsilon}_{\text{bg}}}{f_{\text{an}}\, \omega_0\, A_{\text{dis}}} e^{\frac{Q_{\text{dis}}}{T_0}}\right)^{\frac{1}{n}}. \tag{S17}$$

The characteristic temperature T_c describes the temperature at the start of stress relaxation. Models that have reached LTP undergo heating until deformation transitions from LTP-dominated to dislocation creep-dominated. This transition temperature can be obtained by equating the dislocation creep stress (eq. (S17)) to σ_b and rearranging for temperature, which yields

$$T_\mathrm{t} = \frac{Q_\mathrm{dis}}{\ln\left(\dfrac{\sigma_\mathrm{b}^n\, f_\mathrm{an}\, \omega_0\, A_\mathrm{dis}}{\dot{\varepsilon}_\mathrm{bg}}\right)}. \tag{S18}$$

Models that enter relaxation directly after elastic loading (Figure 2b,d) have undergone no significant heating. Therefore, we define characteristic temperature as

$$T_\mathrm{c} = \max\left(T_0, T_\mathrm{t}\right). \tag{S19}$$

If $T_0 > T_\mathrm{t}$, the model enters stress relaxation, directly after elastic loading at $\tau_\mathrm{c} = \tau_{\mathrm{dis},0}$, and if $T_\mathrm{t} > T_0$, the model reaches the LTP limit and heats up until reaching T_t and $\tau_\mathrm{c} = \sigma_\mathrm{b}$ from where it enters relaxation.

We define the characteristic time as the Maxwell relaxation time of the host rock at characteristic temperature and stress

$$t_\mathrm{c} = t_\mathrm{r} = \frac{\tau_\mathrm{c}}{2\,\dot{\varepsilon}_\mathrm{host}\, G\, f_\mathrm{an}}. \tag{S20}$$

$\dot{\varepsilon}_\mathrm{host}$ is the strain rate in the host rock which can be defined analogously to equations (S14) and (S15):

$$\dot{\varepsilon}_\mathrm{host} = A_\mathrm{dis}\, e^{-\frac{Q_\mathrm{dis}}{T_\mathrm{c}}}\, \tau_\mathrm{c}^n = \frac{\dot{\varepsilon}_\mathrm{an}}{\omega_0} = \frac{\dot{\varepsilon}_\mathrm{bg}}{f_\mathrm{an}\, \omega_0}. \tag{S21}$$

This yields

$$t_{\mathrm{c}} = t_{\mathrm{r}} = \frac{\tau_{\mathrm{c}}\,\omega_0}{2\,\dot{\varepsilon}_{\mathrm{bg}}\,G}. \tag{S22}$$

The characteristic length is defined as the full-width-half-maximum of the anomaly

$$l_{\mathrm{c}} = h. \tag{S23}$$

We can now insert t_{c} and l_{c} into equation (S4), which results in

$$\frac{\partial T'}{\partial t'} = \frac{\tau_{\mathrm{c}}\,\omega_0}{2\,\dot{\varepsilon}_{\mathrm{bg}}\,G}\frac{\kappa}{h^2}\frac{\partial^2 T'}{\partial x'^2} + \frac{1}{\rho\,C_p\,T_{\mathrm{c}}}\frac{\tau_c^2\,\omega_0}{2\,\dot{\varepsilon}_{\mathrm{bg}}\,G}\tau'\dot{\varepsilon}_{\mathrm{vi}}. \tag{S24}$$

Based on assumptions (i) and (ii), $\dot{\varepsilon}_{\mathrm{vi}}$ can be expressed as

$$\dot{\varepsilon}_{\mathrm{vi}} = \omega(x)\,A_{\mathrm{dis}}\,e^{-\frac{Q_{\mathrm{dis}}}{T}}\,\tau^n, \tag{S25}$$

which can be rewritten as

$$\dot{\varepsilon}_{\mathrm{vi}} = \omega(x)\,A_{\mathrm{dis}}\,e^{-\frac{Q_{\mathrm{dis}}}{T}+\frac{Q_{\mathrm{dis}}}{T_{\mathrm{c}}}-\frac{Q_{\mathrm{dis}}}{T_{\mathrm{c}}}}\,\tau_{\mathrm{c}}^n\,\tau'^n, \tag{S26}$$

$$\dot{\varepsilon}_{\mathrm{vi}} = A_{\mathrm{dis}}\,e^{-\frac{Q_{\mathrm{dis}}}{T_{\mathrm{c}}}}\,\tau_{\mathrm{c}}^n\;\omega(x)\,\tau'^n\,e^{\frac{Q_{\mathrm{dis}}}{T_{\mathrm{c}}}-\frac{Q_{\mathrm{dis}}}{T}}, \tag{S27}$$

$$\dot{\varepsilon}_{\mathrm{vi}} = A_{\mathrm{dis}}\,e^{-\frac{Q_{\mathrm{dis}}}{T_{\mathrm{c}}}}\,\tau_{\mathrm{c}}^n\,\omega(x)\,\tau'^n\,e^{\frac{Q_{\mathrm{dis}}}{T_{\mathrm{c}}}\frac{T'-1}{T'}}. \tag{S28}$$

Using equations (S14) and (S15), this can be simplified to

$$\dot{\varepsilon}_{\mathrm{vi}} = \frac{\dot{\varepsilon}_{\mathrm{bg}}}{f_{\mathrm{an}}\,\omega_0}\,\omega(x)\,\tau'^n\,e^{\frac{Q_{\mathrm{dis}}}{T_{\mathrm{c}}}\frac{T'-1}{T'}}. \tag{S29}$$

Substituting equation (S29) into (S24) yields

$$\frac{\partial T'}{\partial t'} = \underbrace{\frac{\tau_\mathrm{c}\,\omega_0}{2\,\dot{\varepsilon}_\mathrm{bg}\,G}}_{t_\mathrm{r}}\,\underbrace{\frac{\kappa}{h^2}}_{t_\mathrm{d}^{-1}}\,\frac{\partial^2 T'}{\partial x'^2} + \underbrace{\frac{1}{\rho\,C_p\,T_\mathrm{c}}}_{u_\mathrm{th}^{-1}}\,\underbrace{\frac{\tau_c^2}{2\,G\,f_\mathrm{an}}}_{u_\mathrm{el}}\,\omega(x)\,\tau'^{n+1}\,e^{\frac{Q_\mathrm{dis}}{T_\mathrm{c}}\frac{T'-1}{T'}}. \tag{S30}$$

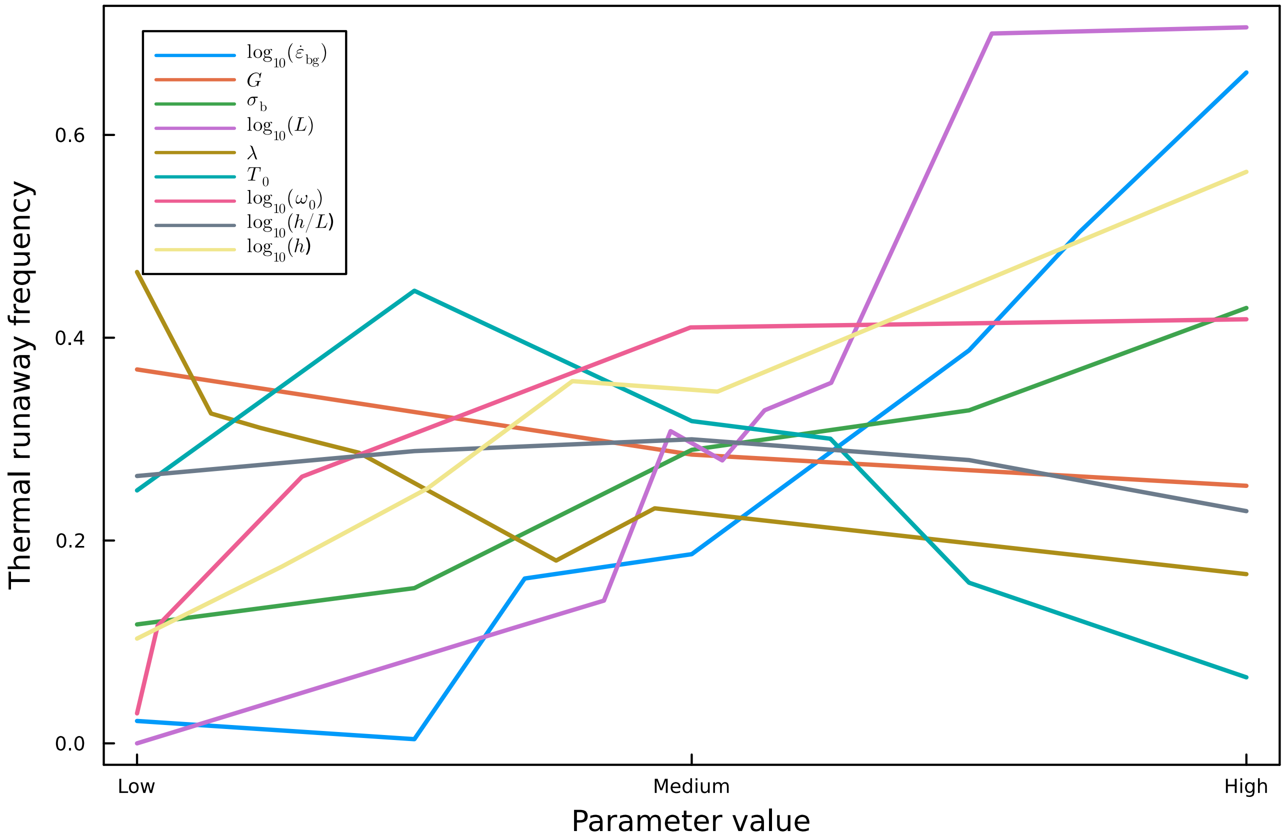

Figure S1. Fraction of models with thermal runaway for each input parameter. All Parameters are scaled with regards to the minimum and maximum value we used (Table 2). For some parameters, we scaled the logarithm to the base 10 of the parameter for convenience. This display is biased by the fact that not every single parameter combination exists but illustrates the general influence and linear/nonlinear behavior of parameters regardless. Only values used at least 150 times are displayed.

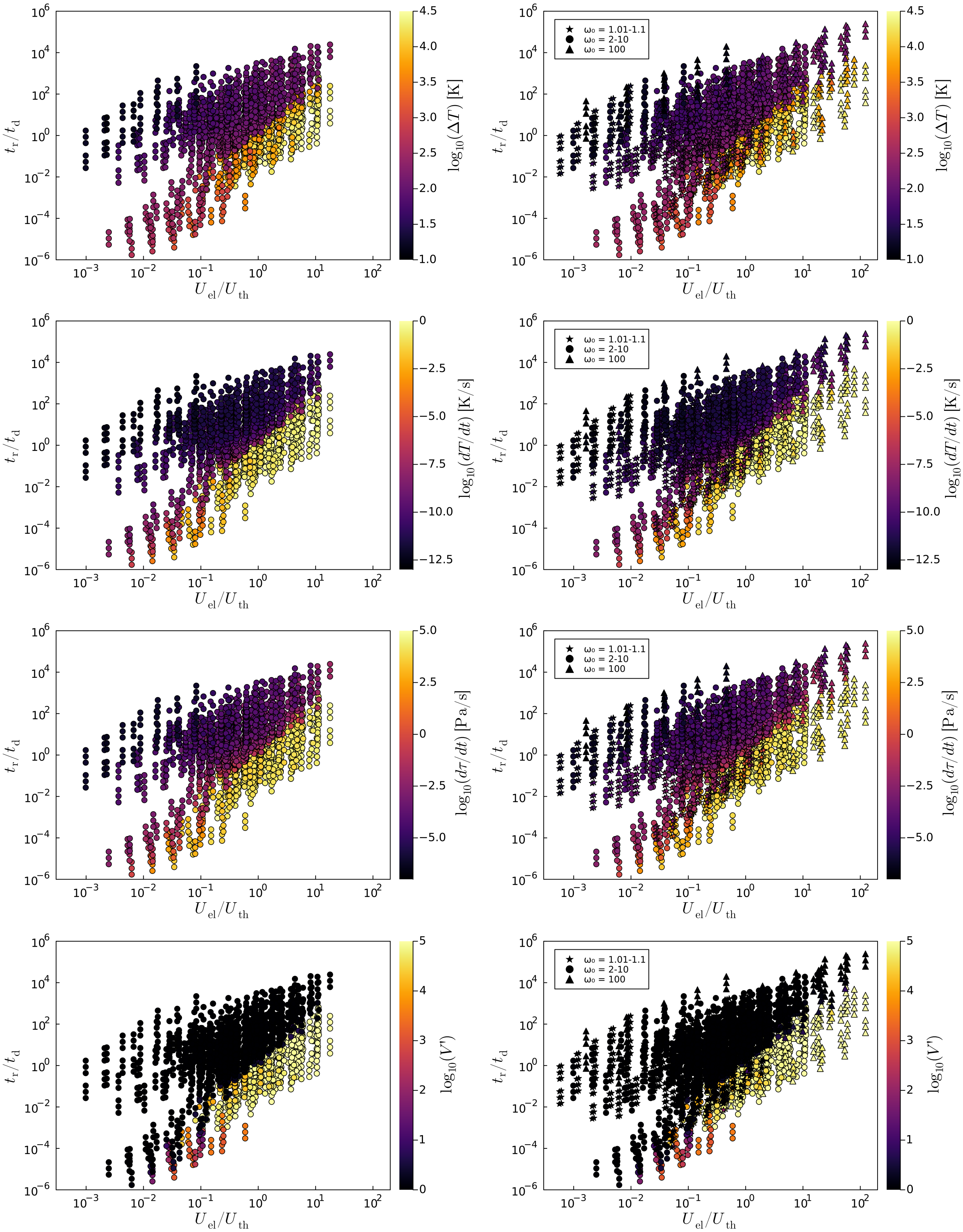

Figure S2. Different metrics to map thermal runaway as functions of t_r/t_d and U_{el}/U_{th}. 1st row: maximum temperature change. 2nd row: maximum temperature gradient. 3rd row: maximum stress gradient. 4th row: maximum velocity divided by boundary velocity. Left column: $\omega_0 = 2 - 10$. Right column: all models.

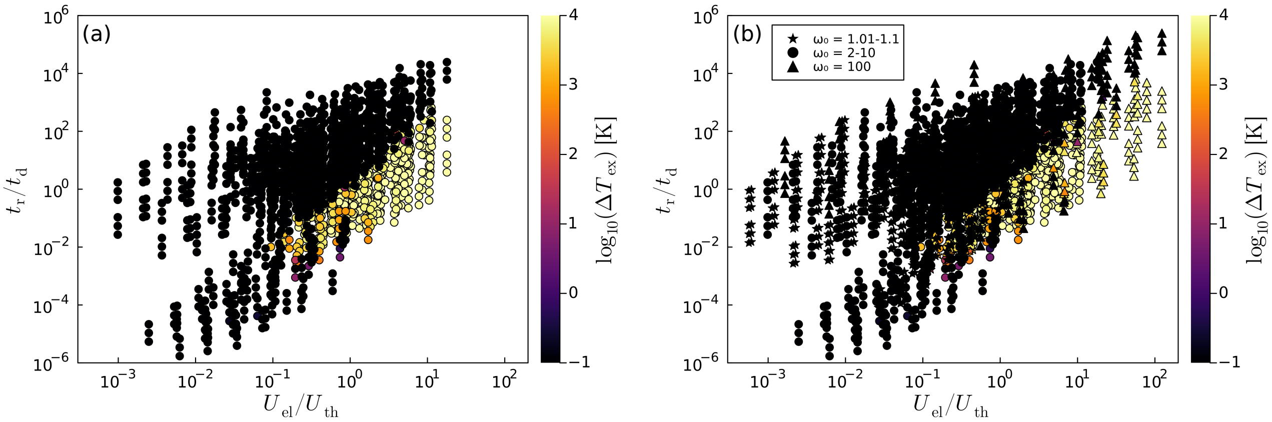

Figure S3. Maximum excess temperature rise ΔT_{ex} as a function of two non-dimensional parameters for step-like anomaly. $t_{\mathrm{r}}/t_{\mathrm{d}}$ denotes the relation between the stress relaxation time scale and the heat diffusion time scale. $U_{\mathrm{el}}/U_{\mathrm{th}}$ denotes the ratio between elastic and thermal energy at the start of stress relaxation. (a) Models with $\omega_0 = 2 - 10$. (b) All models. Note that the colorbar is truncated towards low values.